cost and R&D marginal return are equalized[14] (David, Hall and Toole, 2000, p. 504).

The background of analytical models refers to a "different-from-externality" problem for private R&D investment: the gap between the private rate of return and the cost of capital, when the investing firm and the investors are different entities (see also Hall, 2002)[15]; the opportunity cost of capital is described by an upward sloping schedule where, even if the firm uses only retained earnings for funding R&D investment, at the margin, when R&D investments increase, firm has to look for external investors for other projects (for instance tangible capital acquisition); in fact, financing R&D outlay by using capital from sources such as equity or debt is more costly. In other words, different structures of the R&D capital cost schedule and of the pre-grant optimal level of R&D investment, can explain different firm reactions to a given public subsidy.

If the hypothesis of a "marginal" demand of public fund is retained, the neutrality effect can be the case of a firm with an inelastic (vertical) R&D capital cost curve (that is, an asset constrained firm): in this case the public grant shifts the cost curve to the right exactly of the grant amount[16].

The (partial) crowding-out can be the case of a firm with an upward sloping R&D marginal cost curve, where the public grant produces an increase in R&D expenditure lower than the grant amount substituting for more costly external sources; the elasticity of R&D expenditure changes along the capital cost curve: the more distant the optimal pre-grant level of R&D is from investment funded by "internally generated funds", the higher the crowding-out effect.

Positive cases, therefore, can be identified when at the pre-grant equilibrium a firm's marginal return curve cuts the marginal cost curve in the horizontal portion (where internal fund are available, at a constant cost of capital), or in a portion of the cost sloping curve where elasticity to fund supply is still high. David, Hall and Toole (2000, p. 507) give this explication: in this case public grants provide a signal for the equity holders and the cost of the firm's fund is shifted down, with a substantial increase of R&D investment; thus, the higher the amount of the grant, the more important the signal effect[17]. The effect on non public source of fund can also be different by country: for instance in the US the grant agency review of R&D proposals represents a trustable certification and can positively impact on the total spending of grant recipients[18]. The shape of the R&D capital cost curve depends on the presence of other technological policy measures (fiscal treatment of R&D expenditure or of capital gain) and on the cost of private funding.

As to the marginal return curve, it can be more or less sensible to expected future demand signals or to expected other projects success and it is function of a set of variables such as market conditions, technological opportunities and appropriability conditions.

Few econometric experiments make use of a complete analytical approach; generally, very simplified structural models have been provided such as, for instance, that of Lichtenberg (1987) estimating the following reduced form equation from a system of equations reminding to the David, Hall and Toole (2000) model:

$$R^* = h(\mathbf{X}, \mathbf{Z})$$

where R^* is the private R&D expenditure, expressed as function of variables **X** related to R&D cost (such as public subsidy) and variables **Z** related to R&D returns (such as firm sales); this "early" model did not deal with the problem of endogeneity (see below), assuming the policy variable as strictly exogenous.

[14] A firm derives its R/D marginal return curve when "rationally considers the expected cost and benefit streams for each project", given the technological innovation possibility set (David, Hall and Toole, 2000, p. 503).

[15] The unitary cost of R/D capital is based on a R/D investment fund demand and supply and it increases with the level of R/D and the type of source of funds.

[16] A case of asset constrained firm could be that of firms with a high ratio between the cost of other internal functions (design, engineering, marketing, commercialisation) and the cost of the R/D internal function (Stead, 1976); in these cases firms are more cautious in positively reacting to public subsidies.

[17] Indeed, the additional financial input could be used for other kind of investments, since R/D projects includes a premium to marginal cost for the risk and are in competition with other projects within a firm.

[18] See Diamond (1998) and Jaffe (2002) cited in Lööf and Heshmati (2005, p. 5).