Given the independence assumption, the joint density function for u and v is the product of their individual density functions, and so

$$f_{U,V}(u,v) = \frac{\exp-\frac{1}{2}\{[(u-\delta' z)^2/\sigma_u^2]+v^2/\sigma_v^2\}}{2\pi\sigma_u\sigma_v\Phi[\delta' z/\sigma_u]}; \qquad \text{[A.5]}$$

since $\psi = v + u$, the joint density function for ψ and u is

$$f_{\Psi,U}(\psi,u) = \frac{\exp-\frac{1}{2}\{[(\psi-u)^2/\sigma_v^2]+[(u-\delta' z)^2/\sigma_u^2]\}}{2\pi\sigma_u\sigma_v\Phi[\delta' z/\sigma_u]}, \quad u \ge 0$$

$$= \frac{\exp-\frac{1}{2}\{[(u-\mu_*)^2/\sigma_*^2]+(\psi^2/\sigma_v^2)+(\delta' z/\sigma_u)^2-(\mu_*/\sigma_*)^2\}}{2\pi\sigma_u\sigma_v\Phi[\delta' z/\sigma_u]}, \quad \text{[A.6a]}$$

or, alternatively,

$$f_{\Psi,U}(\psi,u) = \frac{\exp-\frac{1}{2}\{[(u-\mu_*)^2/\sigma_*^2]+[(\psi-\delta' z)^2/(\sigma_v^2+\sigma_u^2)]\}}{2\pi\sigma_u\sigma_v\Phi[\delta' z/\sigma_u]}, \qquad \text{[A.6b]}$$

where

$$\mu_* = \frac{\sigma_v^2\delta' z+\sigma_u^2\psi}{\sigma_v^2+\sigma_u^2} \qquad \text{[A.7]}$$

and

$$\sigma_*^2 = \sigma_v^2\sigma_u^2/(\sigma_v^2+\sigma_u^2). \qquad \text{[A.8]}$$

Thus the marginal density function for $\psi = v + u$ is obtained by integrating u out of $f_{\Psi,U}(\psi,u)$, which yields

$$f_{\Psi}(\psi) = \frac{\exp-\frac{1}{2}\{(\psi^2/\sigma_v^2)+(\delta' z/\sigma_u)^2-(\mu_*/\sigma_*)^2\}}{\sqrt{2\pi}\sigma_u\sigma_v\Phi[\delta' z/\sigma_u]}\int_0^\infty \frac{\exp-\frac{1}{2}\{(u-\mu_*)^2/\sigma_*^2\}}{\sqrt{2\pi}}\,du$$

$$= \frac{\exp-\frac{1}{2}\{(\psi^2/\sigma_v^2)+(\delta' z/\sigma_u)^2-(\mu_*/\sigma_*)^2\}}{\sqrt{2\pi}(\sigma_u^2+\sigma_v^2)^{1/2}\{\Phi[\delta' z/\sigma_u]/\Phi[\mu_*/\sigma_*]\}}, \qquad \text{[A.9a]}$$