or, alternatively,

$$f_{\Psi}(\psi) = \frac{\exp - \frac{1}{2}\{(\psi - \delta' z)^2 / (\sigma_v^2 + \sigma_u^2)\}}{\sqrt{2\pi}(\sigma_u^2 + \sigma_v^2)^{1/2}\{\Phi[\delta' z / \sigma_u] / \Phi[\mu_* / \sigma_*]\}}. \qquad \text{[A.9b]}$$

The density function for the cost value, vc_{ft}, in equation [A.1], is most conveniently given using the expression in equation [A.9b],

$$f_{VC_{ft}}(vc_{ft}) = \frac{\exp - \frac{1}{2}\left\{\frac{[vc_{ft} - vc(x_{ft};\beta) - \delta' z]^2}{\sigma_v^2 + \sigma_u^2}\right\}}{\sqrt{2\pi}(\sigma_u^2 + \sigma_v^2)^{1/2}\{\Phi[d_{ft}] / \Phi[d_{ft}^*]\}}, \qquad \text{[A.10]}$$

where $d_{ft} = \delta' z_{ft} / \sigma_u$, $d_{ft}^* = \mu_{ft}^* / \sigma_*$ and $\mu_{ft}^* = [\sigma_v^2 \delta' z_{ft} + \sigma_u^2 (vc_{ft} - vc(x_{ft};\beta)] / (\sigma_u^2 + \sigma_v^2)$.

Given that there are T_f observations obtained for the f^{th} firm, where $1 \le T_f \le T$, and $vc_f \equiv (vc_{f1}, vc_{f2}, ..., vc_{fT_f})'$ denotes the vector of the T_f cost values in equation [A.1], then the logarithm of the likelihood function for the sample observations, $vc \equiv (vc_1', vc_2', \ldots, vc_F')'$, is

$$\begin{aligned} L(\Theta^*; vc) = &-\frac{1}{2}\left(\sum_{f=1}^{F} T_f\right)\left\{\ln 2\pi + \ln(\sigma_u^2 + \sigma_v^2)\right\} \\ &-\frac{1}{2}\sum_{f=1}^{F}\sum_{t=1}^{T_f}\left\{[vc_{ft} - vc(x_{ft};\beta) - \delta' z_{ft}]^2 / (\sigma_u^2 + \sigma_v^2)\right\} \\ &-\frac{1}{2}\sum_{f=1}^{F}\sum_{t=1}^{T_F}\left\{\ln\Phi[d_{ft}] - \ln\Phi[d_{ft}^*]\right\}, \end{aligned} \qquad \text{[A.11]}$$

where $\Theta^* \equiv (\beta', \delta', \sigma_u^2, \sigma_v^2)'$.

Using the re-parameterization of the model suggested by Battese and Corra (1977), involving the parameters $\sigma^2 \equiv (\sigma_v^2 + \sigma_u^2)$ and $0 \le \gamma \equiv \sigma_u^2/(\sigma_v^2 + \sigma_u^2) \le 1$, the logarithm of the likelihood function can be expressed by

$$\begin{aligned} L(\Theta; vc) = &-\frac{1}{2}\left(\sum_{f=1}^{F} T_f\right)\left\{\ln 2\pi + \ln\sigma^2\right\} \\ &-\frac{1}{2}\sum_{f=1}^{F}\sum_{t=1}^{T_f}\left\{[vc_{ft} - vc(x_{ft};\beta) - \delta' z_{ft}]^2 / \sigma^2\right\} \\ &-\frac{1}{2}\sum_{f=1}^{F}\sum_{t=1}^{T_f}\left\{\ln\Phi[d_{ft}] - \ln\Phi[d_{ft}^*]\right\}, \end{aligned} \qquad \text{[A.12]}$$