Appendix

The log-likelihood function presented in the Appendix of the Battese and Coelli (1993) working paper refers to a stochastic frontier *production* function, with the u_{ft}*s* interpreted as pure *technical inefficiency* effects, which cause the firm to operate below the production frontier. If we wish to specify a stochastic frontier *cost* function, we have to alter the global error term specification from $\psi_{ft} = (v_{ft} - u_{ft})$, as in Battese and Coelli (1993), to $\psi_{ft} = (v_{ft} + u_{ft})$, as in the equation [2] reported in the text[81]. The u_{ft}*s* now define how far the firm operates above the cost frontier and involve both *technical and allocative inefficiencies*. The log-likelihood function for the cost frontier specification analogue of the Battese and Coelli model can be obtained by making a few simple sign changes and is reproduced here.

For simplicity of presentation of results in this Appendix, we assume that the stochastic frontier cost model [1]-[2] is expressed by

$$vc_{ft} = vc(x_{ft};\beta) + \psi_{ft} \qquad \text{[A.1]}$$

$$\psi_{ft} = v_{ft} + u_{ft} \qquad \text{[A.2]}$$

$f = 1, \ldots, F$, and $t = 1, \ldots, T_f$,

where $vc_{ft} = \ln VC_{ft}$, $vc(.) = \ln VC(.)$ and x_{ft} is a vector which groups the arguments of the variable cost function, Y_{ft}, P_{ft}, Z_{ft}, and τ_{ft}; further, $v_{ft} \sim i.i.d.\ N(0, \sigma_v^2)$ and $u_{ft} \sim N^+(\delta' z_{ft}, \sigma_u^2)$.

The density functions for v_{ft} and u_{ft} are

$$f_V(v) = \frac{\exp\left\{-\frac{1}{2} v^2 / \sigma_v^2\right\}}{\sqrt{2\pi}\sigma_v}, \qquad -\infty < v < \infty \qquad \text{[A.3]}$$

and

$$f_U(u) = \frac{\exp\left\{-\frac{1}{2}(u - \delta' z)^2 / \sigma_u^2\right\}}{\sqrt{2\pi}\sigma_u \Phi[\delta' z / \sigma_u]}, \quad u \geq 0, \qquad \text{[A.4]}$$

where the subscripts, *f* and *t*, are omitted for convenience in the presentation; and $\Phi[\cdot]$ is the standard normal cumulative distribution function.

[81] The inefficiency effect, u_{ft}, is *added* in the stochastic cost frontier instead of being *subtracted*, as in the case of the stochastic production frontier, because the cost function represents *minimum cost*, whereas the production function represents *maximum output*.