

Divide now equation (5) by  $p_t$  and by  $\partial D(t)/\partial p_t$  to obtain the following condition that describes the optimal price path:

$$(1 - \tau_t) \left\{ \frac{q(t)}{p_t} \frac{\partial p_t}{\partial D_t} + (1 + \lambda_t) - \frac{\partial c(t)}{\partial q_t} \frac{1}{p_t} (1 + \lambda_t) - \frac{\partial h(t)}{\partial q_t} \frac{1}{p_t} (1 + \lambda_t) \right\} \quad (7)$$

$$- E_t \beta_{t,t+1} (1 - \tau_{t+1}) \left\{ \frac{\partial h(t+1)}{\partial q_t} \frac{1}{p_t} (1 + \lambda_t) + \frac{\partial \tilde{a}(t+1)}{\partial (p_t, q_t)} \left[ \frac{q(t)}{p_t} \frac{\partial p_t}{\partial D_t} + (1 + \lambda_t) \right] b_t \right\} = 0$$

Now let  $(\partial p_t / \partial D_t)(q_t / p_t) = \varepsilon_t$  be the inverse of the direct effect price elasticity and let marginal cost be given by  $\partial c(t) / \partial q_t = \eta_t(c_t / q_t)$ , where  $\eta_t$  denotes the cost elasticity of output, the reciprocal of the scale elasticity. Using these definitions and dividing equation (7) by  $(1 - \tau_t)$  we obtain:

$$\varepsilon_t + (1 + \lambda_t) - \eta_t \frac{c_t}{q_t} \frac{1}{p_t} (1 + \lambda_t) - \frac{\partial h(t)}{\partial q_t} \frac{1}{p_t} (1 + \lambda_t) \quad (8)$$

$$- E_t \beta_{t,t+1} \frac{(1 - \tau_{t+1})}{(1 - \tau_t)} \left\{ \frac{\partial h(t+1)}{\partial q_t} \frac{1}{p_t} (1 + \lambda_t) + \frac{\partial \tilde{a}(t+1)}{\partial (p_t, q_t)} \left[ \varepsilon_t + (1 + \lambda_t) \right] b_t \right\} = 0$$

Observe now that the second and third terms of equation (8) can be rewritten as follows:

$$(1 + \lambda_t) \left( \frac{p_t q_t - c_t}{p_t q_t} \right) + (1 + \lambda_t) (1 - \eta_t) \frac{c_t}{p_t q_t} = (1 + \lambda_t) \left[ PCM_t + (1 - \eta_t) \frac{c_t}{p_t q_t} \right] \quad (9)$$

where  $PCM_t$  is the firm's price-cost margin. Substitute (9) into (8) and divide throughout by  $(1 + \lambda_t)$  to finally obtain the following expression for the firm's price-cost margin: