

The analysis carried out in this work highlights that the results obtained in the empirical literature depend on the choice of the model, consequently on the hypothesis of the underlying technology.

In the case of a single output firm a measure of economies of scale are reflected by the output elasticity of cost:

$$(11) \quad \varepsilon_{C,Y} = \frac{dC}{dY} \frac{Y}{C} = \frac{MC}{AC}$$

The firm experiences increasing, constant or decreasing returns to scale if  $\varepsilon_{c,y}$  is less than, equal to or greater than one.

In the multiproduct case we can consider two distinct measures: ray economies of scale and product-specific economies of scale. The first indicates how total costs increase when every output increases by the same percentage; in formula:

$$\varepsilon_{C,Y}^R = \frac{\sum_i Y_i \frac{\partial C}{\partial Y_i}}{C}$$

Product specific economies of scale measure how costs change with each output, the quantities of the other products being constant. Defining the average incremental cost AIC as the increase in total cost associated with the production of a given output, as compared with not producing it at all, divided by that output, the measure of product specific economies of scale is:

$$\varepsilon_{Y_i}^{PS} = \frac{MC_i}{AIC_i}$$

The inclusion of hedonic variables in the cost function along with physical output permits us to consider different measures of economies of scale. The elasticity of cost with respect to output (here defined economies of output) indicates the increasing in costs when volumes supplied are expanded while keeping hedonic variables fixed. If volumes and number of users are expanded proportionally, the increase in costs associated with the expansion of the firm is measured by:

$$(12) \quad \varepsilon_S = \varepsilon_Y + \varepsilon_{UT}$$