

that is:

$$E\left\{\frac{[w-p(\mathbf{x})]y}{[1-p(\mathbf{x})]}\right\} = E\{w(y_1 - y_0)\}$$

From the LIE 3 we know that, if  $x$  is a generic discrete variable assuming values  $x = (x_1, x_2, \dots, x_M)$  with probabilities  $p = (p_1, p_2, \dots, p_M)$ , then:

$$E(h) = p_1 \cdot E(h | x_1) + p_2 \cdot E(h | x_2) + \dots + p_M \cdot E(h | x_M).$$

Thus, by assuming  $h=w(y_1-y_0)$ , we get that:

$$\begin{aligned} E(h) &= E[w(y_1-y_0)] = p(w=1) \cdot E[w(y_1-y_0) | w=1] + p(w=0) \cdot E[w(y_1-y_0) | w=0] = \\ &= p(w=1) \cdot E[(y_1-y_0) | w=1] = p(w=1) \cdot ATET. \end{aligned}$$

It means that:

$$E\left\{\frac{[w-p(\mathbf{x})]y}{[1-p(\mathbf{x})]}\right\} = E\{w(y_1 - y_0)\} = p(w=1) \cdot ATET$$

proving that:

$$ATET = E\left\{\frac{[w-p(\mathbf{x})]y}{p(w=1)[1-p(\mathbf{x})]}\right\} \quad (5)$$

Now, by remembering that  $ATE = p(w=1) \cdot ATET + p(w=0) \cdot ATENT$ , we have that:

$$\begin{aligned} ATENT &= \frac{ATE}{p(w=0)} - \frac{p(w=1)}{p(w=0)} ATET = \\ &= \frac{1}{p(w=0)} E\left\{\frac{[w-p(\mathbf{x})]y}{p(\mathbf{x})[1-p(\mathbf{x})]} - p(w=1) \frac{[w-p(\mathbf{x})]y}{p(w=1)[1-p(\mathbf{x})]}\right\} = \\ &= \frac{1}{p(w=0)} E\left\{\frac{[w-p(\mathbf{x})]y}{p(\mathbf{x})[1-p(\mathbf{x})]} - \frac{[w-p(\mathbf{x})]y}{[1-p(\mathbf{x})]}\right\} \\ &= \frac{1}{p(w=0)} E\left\{\frac{[w-p(\mathbf{x})]y - p(\mathbf{x})[w-p(\mathbf{x})]y}{p(\mathbf{x})[1-p(\mathbf{x})]}\right\} = \\ &= \frac{1}{p(w=0)} E\left\{\frac{[w-p(\mathbf{x})]y[1-p(\mathbf{x})]}{p(\mathbf{x})[1-p(\mathbf{x})]}\right\} = \frac{1}{p(w=0)} E\left\{\frac{[w-p(\mathbf{x})]y}{p(\mathbf{x})}\right\} = \\ &= E\left\{\frac{[w-p(\mathbf{x})]y}{p(w=0)p(\mathbf{x})}\right\} \end{aligned}$$