



**WORKING PAPERS**

WP.70

**AN INTEGRATED MODEL FOR THE DYNAMIC  
ANALYSIS OF LOCATION-TRANSPORT  
INTERRELATIONS**

*C.S. Bertuglia - G. Leonardi - S. Occelli  
G.A. Rabino - R. Tadei*





Summary

This report describes the current state of development of the  
second phase of the research project "Location-Transport Interrelations"  
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**AN INTEGRATED MODEL FOR THE DYNAMIC  
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The objective of this research project is to develop a dynamic  
model of location-transport interrelations. This is achieved by a  
series of integrated models of the various components. The  
main aim of this research is to provide a theoretical framework  
for the analysis of location-transport interrelations.

At the end of the project an integrated model of location-transport  
interrelations will be developed.

Particular attention has been given to the development of  
the dynamic models of the various components of the location-transport  
interrelations.

Luglio 1985



## Summary

This report describes the current stage of development of the second phase of the research project "Location-transport interrelations" carried out at I.R.E.S. and aimed at building an integrated model of that system.

It describes in particular the general architecture of the system, its various subsystems and their interrelations. This is followed by a more detailed technical description of the subsystems for which the model and software have already been developed.

At the end of the report we give an indication of the problems which still have to be resolved.

Particular emphasis has been given to interaction between dynamic systems and problems of the economic interpretation of location-transport models.





## 1. INTRODUCTION

This is a preliminary report on the present state of progress of the second phase of the IRES research programme "Dynamic analysis of location-transport interrelations". In the first phase various alternative approaches were explored and two fundamental research objectives for the successive phase were established (Bertuglia et al., eds., 1983):

- a. to reformulate, in dynamic terms, the integrated location-transport models;
- b. to introduce explicitly and with theoretical rigour the economic variables and mechanisms lacking in current urban models.

It was decided to adopt the traditional structure for the urban model inspired by economic base theory, as it was considered fundamentally valid, but, instead of the general classic equilibrium equations, dynamic equations have been introduced for the main state variables. In addition, price mechanisms have been introduced into certain subsystems.

Sections 2, 3 and 4 give a description of the general structure of the system and subsystems. Section 5 describes the economic and modelling approaches and mathematical techniques. In particular use has been made of the following:

- a. Master Equations (Weidlich and Haag, 1983) to formulate the dynamic equations relating to the mobility of population (both residential and job mobility);
- b. the dynamic equations of Harris and Wilson (1978) to model the stock of services;

c. the equilibrium conditions (market clearing) of Anas (1982) and random bidding models of Leonardi (1983) to model price and wage formation mechanisms.

The general structure of the model is shown in Fig. 1. The model is based on the following assumptions: (1) The market is characterized by a large number of agents, each of whom is represented by a single agent. (2) The agents are assumed to be rational and to have access to the same information. (3) The agents are assumed to be independent and to act in their own self-interest. (4) The agents are assumed to be homogeneous and to have the same preferences. (5) The agents are assumed to be perfectly informed and to have access to the same information. (6) The agents are assumed to be perfectly rational and to have access to the same information. (7) The agents are assumed to be perfectly independent and to act in their own self-interest. (8) The agents are assumed to be perfectly homogeneous and to have the same preferences. (9) The agents are assumed to be perfectly informed and to have access to the same information. (10) The agents are assumed to be perfectly rational and to have access to the same information.

The general structure of the model is shown in Fig. 1.

1. Supply curve
2. Demand curve
3. Market clearing
4. Equilibrium
5. Price formation

The variables considered relevant for describing the structure of the model are the following:

1. Supply curve
2. Demand curve
3. Market clearing
4. Equilibrium
5. Price formation

Each of the variables considered above has an interpretation in the context of the model. The supply curve represents the willingness to supply at different prices. The demand curve represents the willingness to demand at different prices. The market clearing condition represents the condition that the quantity supplied equals the quantity demanded. The equilibrium price represents the price at which the market clears. The price formation mechanism represents the process by which the equilibrium price is determined.

The general structure of the model is shown in Fig. 1.



## 2. THE MAIN VARIABLES AND SUBSYSTEMS

Although an urban system is a complex and indivisible whole, it is possible to identify a number of families of relatively independent interrelations in the sense that they involve a limited subset of variables. These families of interrelations are the principal subsystems of the urban system. Of course the subsystems referred to are only partially independent, as far as the links between them are weaker than those within each individual system.

The principal subsystems of the urban system are the following (see fig. 1):

1. housing market;
2. job market;
3. service sector;
4. land-market;
5. transport.

The variables considered essential for describing the structure of an urban system are the following:

1. population;
2. housing stock;
3. industry (economic base);
4. services;
5. land-use;
6. traffic flows.

Each of the variables listed above can be disaggregated at different levels according to the data available and the specific requirements. However it is necessary to make at the very least a spatial disaggregation, i.e. a specification of value for each zone into which the urban area has been divided.

The principal families of interrelations which link the above variables are shown in fig. 1.

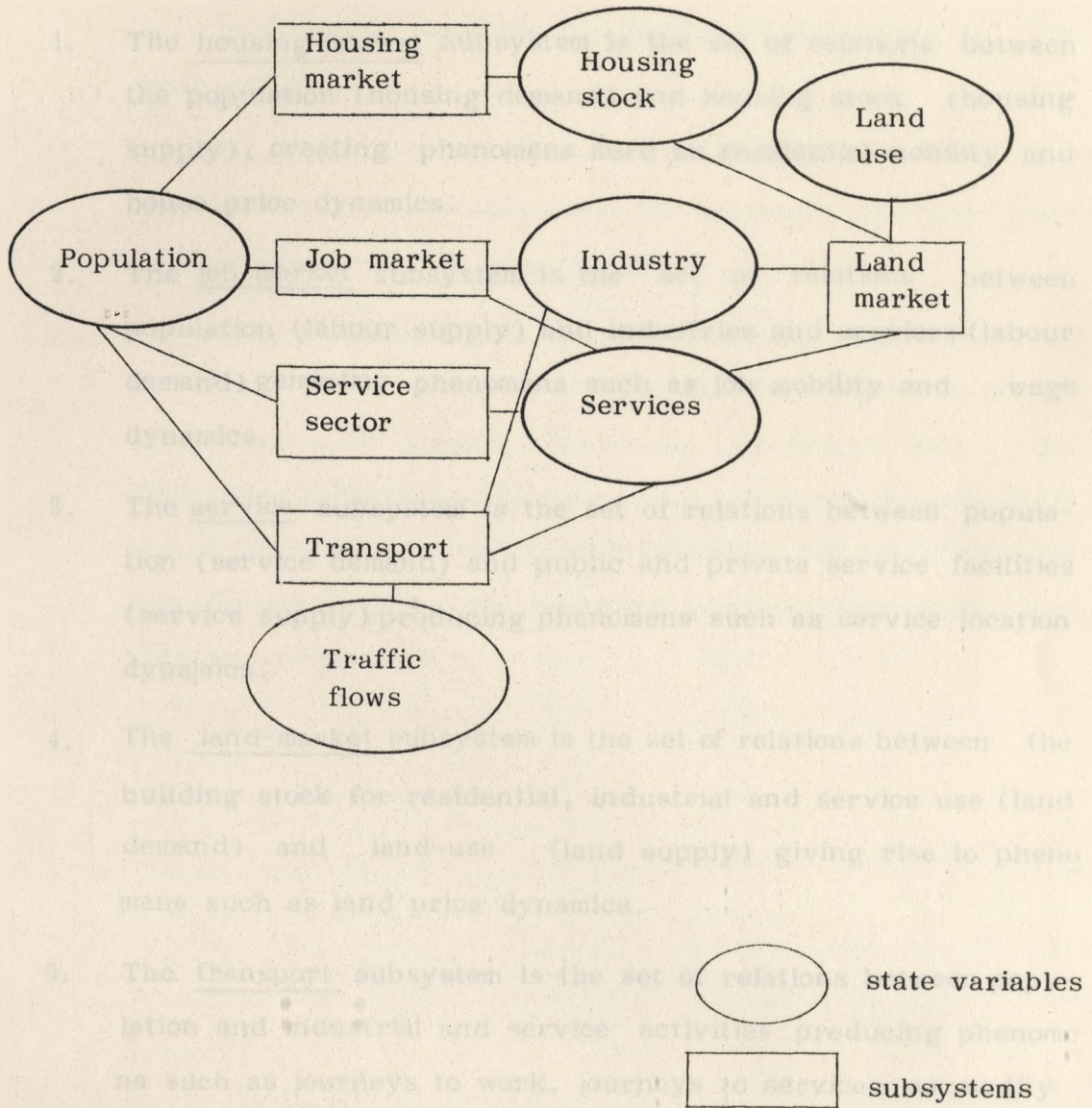


Figure 1 - Main subsystems of an urban system and interrelations with main state variables

1. The housing market subsystem is the set of relations between the population (housing demand) and housing stock (housing supply), creating phenomena such as residential mobility and house price dynamics.
2. The job market subsystem is the set of relations between population (labour supply) and industries and services (labour demand) generating phenomena such as job mobility and wage dynamics.
3. The service subsystem is the set of relations between population (service demand) and public and private service facilities (service supply) producing phenomena such as service location dynamics.
4. The land-market subsystem is the set of relations between the building stock for residential, industrial and service use (land demand) and land-use (land supply) giving rise to phenomena such as land price dynamics.
5. The transport subsystem is the set of relations between population and industrial and service activities producing phenomena such as journeys to work, journeys to services, commodity flows, modal split and network assignment.

Before moving on to a detailed description of these subsystems the following observations are necessary:

- a. the stock dynamics, which is considered endogenous to the housing market and the service sector (both demand-oriented activities), is on the other hand exogenous to the job market. In fact service stock is determined by the service subsystem and industry stock, according to ur-



ban modelling tradition, is considered exogenous to the whole system;

- b. the land market will not be dealt with explicitly. It is assumed that in each subsystem the necessary land-use constraints are arrived at exogenously;
- c. the transport subsystem is also not dealt with explicitly. In effect the basic component of the transport subsystem, i.e. the generation of transport demand, is so closely interlinked with the first three subsystems (housing, jobs and services) that it is one of their principal endogenous variables. Well established models already exist for the remaining aspects (modal split and network assignment).

### 3. DESCRIPTION OF SUBSYSTEMS

#### 3.1. The housing market

The housing market consists of three basic phenomena:

- a. residential mobility. This process includes moves of the residential population and changes of a purely demographic nature. While the parameters describing the latter can be assumed exogenously, the factors which induce the population to move are partly endogenous to the housing market itself and partly produced by other subsystems. The endogenous factors are the housing stock, which makes available the residential alternatives and housing prices which define the utilities of different housing choices. The factors produced by other subsystems are work place accessibility (depending on the job market subsystem) and service accessibility (depending on the service subsystem);
- b. housing stock dynamics. The principal exogenous inputs are possible local authority intervention and the availability of residential land. The main endogenous factors are the demand for and price of housing which determine new investments in house building, renewal and demolition;
- c. housing price dynamics. The price mechanism is basically endogenous to the housing market subsystem being an internal signal of the equilibrium between demand and supply. However, form of rent control, public dwellings, subsidies and grants are also to be considered as exogenous inputs. In this case prices become insufficient as indicators of the demand/supply equilibrium and other signals may appear in the

form of externalities, for example queuing and waiting times for access to public and subsidized housing markets.

The above structure can be summarized as in fig. 2. The outputs of this subsystem to other subsystems are not shown and will be discussed as inputs in a detailed analysis of these subsystems. The main output however of the housing market is the population distributed in the residential locations.

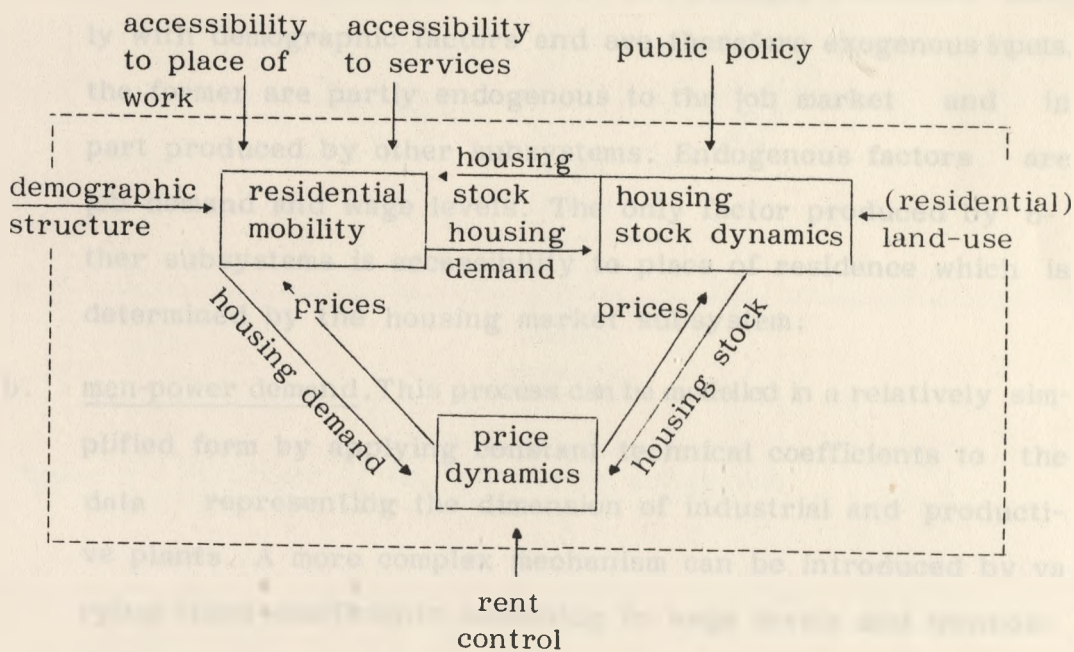


Figure 2 - Structure of the housing market subsystem



### 3.2. Job market

The structure of this subsystem consists of three main phenomena:

- a. job mobility. This includes changes in job location of the working population, transitions from employment to unemployment and vice versa, entry into the job market (first job) and exit (retirement). While the latter two are changes associated mainly with demographic factors and are therefore exogenous inputs, the former are partly endogenous to the job market and in part produced by other subsystems. Endogenous factors are job demand and wage levels. The only factor produced by other subsystems is accessibility to place of residence which is determined by the housing market subsystem;
- b. man-power demand. This process can be modelled in a relatively simplified form by applying constant technical coefficients to the data representing the dimension of industrial and productive plants. A more complex mechanism can be introduced by varying these coefficients according to wage levels and technological (and organisational) innovation in the productive process. This latter is an exogenous input. There is an input from other subsystems (size and location of industries and services) and an endogenous factor (wages);
- c. wage dynamics. The wage formation mechanism is in part endogenous, i.e. it depends on free bargaining between supply and demand of manpower and partly exogenous, i.e. to the extent to which it is controlled by union or public intervention.

The structure described above can be summarized in the following diagram.

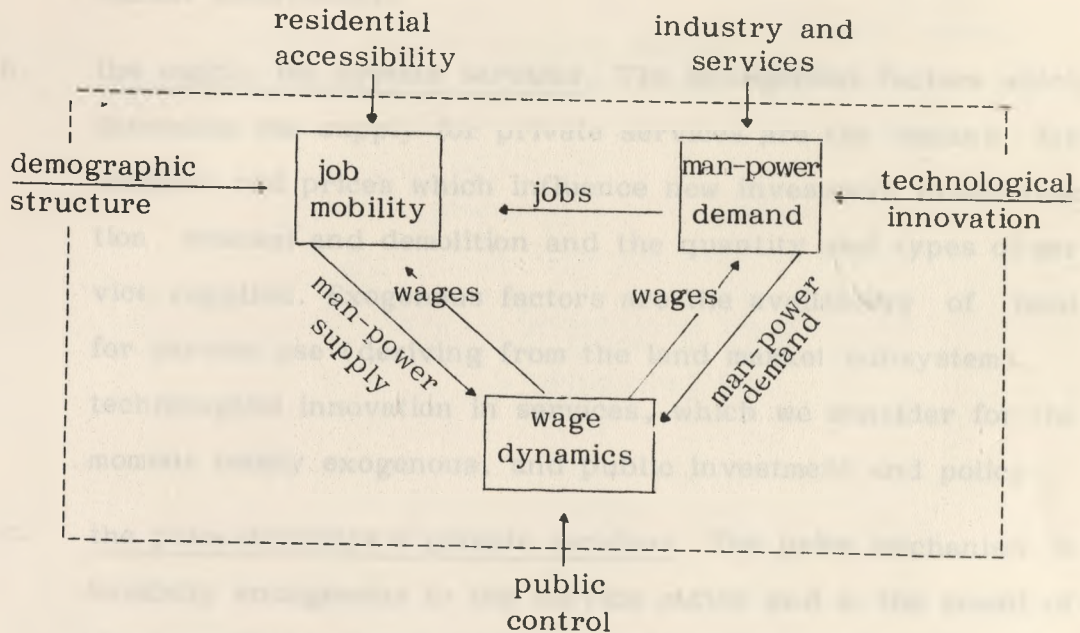


Figure 3 - Structure of the job market subsystem

### 3.3. Service sector

The service sector can be divided into two sectors, public and private which behave differently.

The structure of the private service subsector consists of three basic phenomena:

- a. the demand for private services. The factors determining the demand for private services are partly endogenous to the service subsystem (service supply and prices) and partly produ-

ced by other subsystems i.e. resident population (from the housing market subsystem) and number of jobs (from the job market subsystem);

- b. the supply for private services. The endogenous factors which determine the supply for private services are the demand for services and prices which influence new investment in construction, renewal and demolition and the quantity and types of service supplied. Exogenous factors are the availability of land for service use (deriving from the land market subsystem), technological innovation in services, which we consider for the moment totally exogenous, and public investment and policy;
- c. the price dynamics of private services. The price mechanism is basically endogenous to the service sector and is the result of the balancing between demand and supply. There can be nevertheless an exogenous input in the form of price control when maximum prices are imposed for certain goods and services.

The structure of the private service sector subsystem is summarised in fig. 4.

The comments already made about the private service sector hold also for the public service sector with the exception of the price dynamics which does not apply, due to the nature of the services (schools, health services etc.). There are however other signals which act as equilibrators (eg. waiting or queuing times for services).



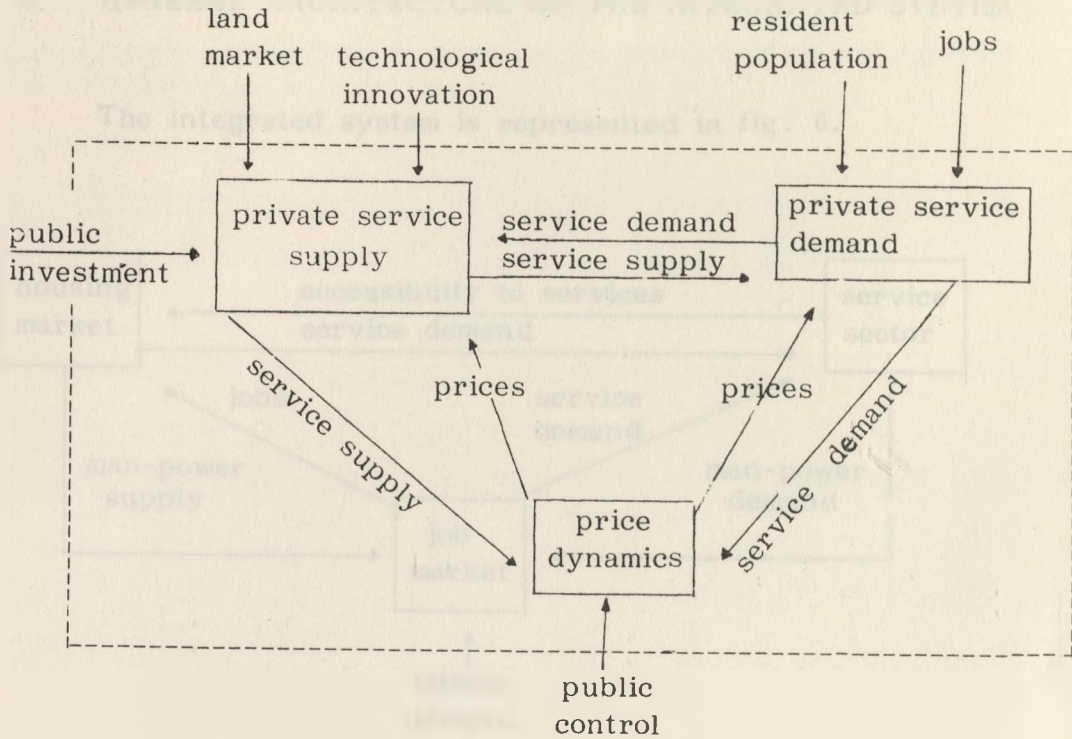


Figure 4 - Structure of the private service sector subsystem

The structure of the public service sector subsystem can be summarized as follows:

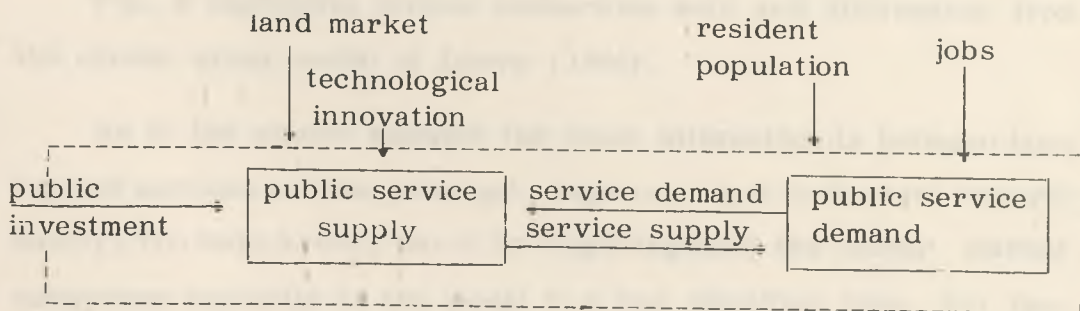


Figure 5 - Structure of the public service sector subsystem

#### 4. GENERAL ARCHITECTURE OF THE INTEGRATED SYSTEM

The integrated system is represented in fig. 6.

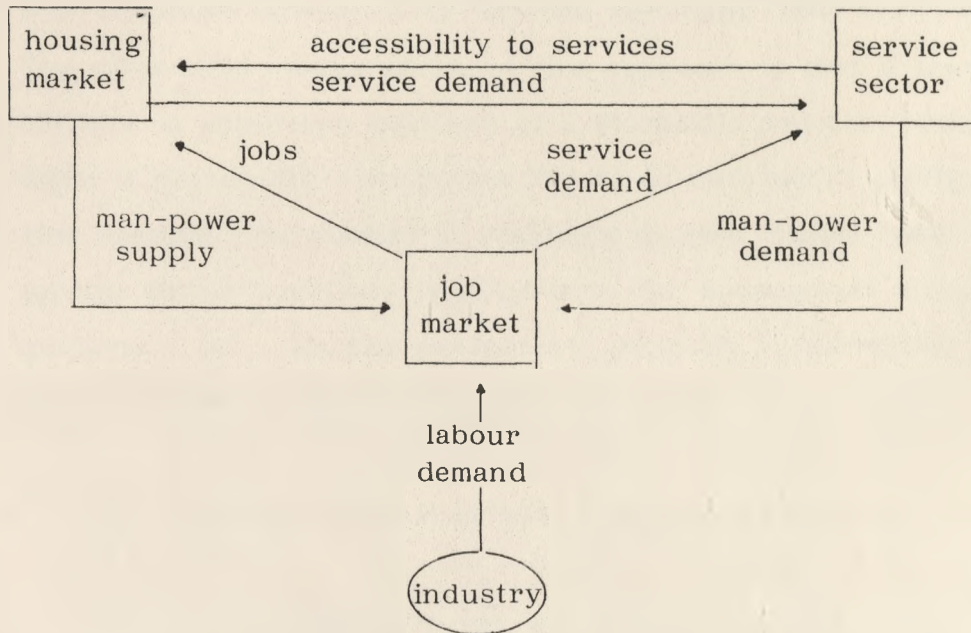


Figure 6 - Framework of the integrated system

Fig. 6 highlights certain similarities with and differences from the classic urban model of Lowry (1964).

As in the classic example the basic interaction is between housing and services and the principal exogenous input is the base sector (industry). The main novelty lies in having integrated the labour market subsystem explicitly in the model in a less simplified form. But the most important difference is the fact that the system in fig. 6 is dynamic. This makes a profound difference with respect to a general equilibrium model. The balancing of supply and demand takes time and supply can be in excess or in shortage. These disequilibria

reflect the inertiae present in processes such as residential or job changes. Clearly in a static model such inertiae are ignored.

The model is composed of residential and job changes. The residential changes are modeled by the residential change equation. The job changes are modeled by the job change equation. The residential change equation is a first-order difference equation. The job change equation is a first-order difference equation. The residential change equation is a first-order difference equation. The job change equation is a first-order difference equation.

$$\frac{dY}{dt} = \dots$$

The crucial element in the use of this approach is the specification of the wage function. The wage function is a function of the level of education. The wage function is a function of the level of education. The wage function is a function of the level of education.

In general, the complete dynamic system of the model requires the use of a computer. The complete dynamic system of the model requires the use of a computer. The complete dynamic system of the model requires the use of a computer.

The differential equations for the residential and job changes are given by the following equations. The differential equations for the residential and job changes are given by the following equations.

$$\dots$$



## 5. MODELLING THE DIFFERENT SUBSYSTEMS

### 5.1. General accounting scheme for population mobility

To model the processes of residential and job mobility, the master equations developed by Weidlich and Haag (1983) have been applied. The distinctive characteristic of this approach is that it treats the changes in population patterns as a stochastic process, which associates a probability distribution  $P(\underline{n};t)$  to each macro-configuration (for example, the number of residents in each place) and using suitable motion equations (analogous to the Kolmogoroff differential equations for Markov processes) provides a description of changes in the probability distribution over time:

$$\frac{d P(\underline{n};t)}{dt} = \sum_{\{k\}} [w_t(\underline{n}, \underline{n+k}) P(\underline{n+k};t) - w_t(\underline{n+k}, \underline{n}) P(\underline{n};t)] \quad (1)$$

The crucial element in the use of this approach is the specification of the ways in which the changes in configuration occur (eg. the ways in which individuals decide to move). This is the problem of the specification of the transition probabilities of the system,  $w_t(\underline{n}, \underline{n+k})$ .

In general the complete stochastic version of the master equations is not used in applications but a deterministic approximation, i.e. the equivalent dynamic equations for the expected values. This approximation is justified by the use of the mean value theorem and the empirically based assumption that the population is aggregated into sufficiently large samples.

The differential equations for the expected values have the general form:

$$\dot{P}_j(t) = Y_j(t) + \sum_i P_i(t) a_{ij}(t) - P_j(t) \sum_i a_{ji}(t) \quad (2)$$

When applied to residential mobility these equations can be read as follows (i and j are the indices associated with the zones of residence):

$P_i(t)$  expected population resident in i at time t;

$a_{ij}(t)$  rate of transition from i to j at time t;  $a_{ij}(t)\Delta$  is the probability that an individual resident in i at time t moves to j in the interval (t, t+ $\Delta$ );

$Y_i(t)$  net population changes and net migrations to and from the rest of the world, in i at time t; i.e.  $Y_i(t)\Delta$  is the population change in i in the interval (t, t+ $\Delta$ ) due to births, deaths, immigration and emigration into and from the urban area.

Equations (2) can be resolved integrating from initial values of  $P_i$  at time t=0 given the exogenous input Y(t) and the transition rates  $a_{ij}(t)$ . Y(t) can be obtained with standard demographic and econometric techniques.

As we stated previously, the functional specification of the transition rates is fundamental. We have chosen the form:

$$a_{ij}(t) = h_{ij} \exp [u_j(t) - u_i(t)], \quad h_{ij} > 0, \quad (3)$$

in which:

$h_{ij}$  is a factor associated with the friction in moving from i to j (and may depend for instance on the distance and cost of the move);

$u_i(t)$  is the expected utility for a resident in  $i$  at time  $t$ .

Rates of transition of the type given by equation (3) are proposed in general terms in Weidlich and Haag (1983) and are justified in micro-economic terms by Leonardi (1983).

Although in equation (3) the utilities  $u_i(t)$  are indicated as explicit functions of time, in this model they are in reality specified as functions of other endogenous and exogenous variables. For the housing market these are:

- a. the resident population;
- b. the stock of vacant housing;
- c. the accessibility to services;
- d. the accessibility to jobs;
- e. the price of houses.

As we have already stated above  $u_i$  is considered a linear function of these variables.

From the computational point of view, it is more convenient to operate with a discrete version of equations (2). The version adopted is:

$$P_j(t+\Delta) = P_j(t) + \Delta Y_j(t) + \Delta \sum_i P_i(t) q_{ij}(t, \Delta) - \Delta P_j(t) \sum_i q_{ij}(t, \Delta) \quad (4)$$

in which:

$$q_{ij}(t, \Delta) = \frac{h_{ij} e^{u_j(t)}}{[\Delta \sum_j h_{ij} e^{u_j(t)}] + e^{u_i(t)}} \quad (5)$$



It can be immediately seen that:

$$\lim_{\Delta \rightarrow 0} q_{ij}(t, \Delta) = h_{ij} \exp [u_j(t) - u_i(t)] = a_{ij}(t)$$

and that (4) therefore tends to (2) as  $\Delta \rightarrow 0$ . The form proposed in equation (5) is a Logit model (Domencich, McFadden, 1975), for which there is a precise micro-economic interpretation as demand behaviour model in the choice between discrete alternatives. (This also justifies in part, a posteriori, the assumption made on the form of the transition rates). The use of these models for the analysis of residential mobility is discussed in greater detail by Anas (1982).

Job mobility can be described with equations analogous to (2), when it is not intended to refer contemporarily to residential mobility. The generalisation which takes into consideration both residential and job mobility simultaneously is formulated by introducing two indices into the description of the state of the population instead of one i.e. the place of residence and the place of work.

The state variables therefore become:

$P_{ij}^i(t)$  expected population resident in  $j$  with a job in  $i$ .

The equations which extend (2) incorporating the changes in place of work (and taking the unemployed into account), written in the form suggested by Leonardi (1983) are:

a) for the employed  $i \neq 0$

$$P_{ij}^i(t+1) = \sum_k P_{ik}^i(t) [q_{kj}^i(t) + p_{kj}^i(t)] + P_{ij}^i(t) [1 - \sum_k q_{jk}^i(t) - \sum_k p_{jk}^i(t)] \quad (6)$$

b) for the unemployed  $i=0$

$$P_{oj}(t+1) = \sum_k P_{ok}(t) q_{kj}^0(t) + \sum_{\substack{ik \\ i>0}} P_{ij}(t) p_{jk}^i(t) + P_{oj}(t) [1 - \sum_k q_{jk}^0(t) - (1 - e^{-\phi_j})] \quad (7)$$

where

$q_{kj}^i(t), i>0$  is the probability that a worker with job in  $i$  and residence in  $k$  at time  $t$  moves his residence to  $j$  at time  $(t+1)$ ;

$q_{kj}^0(t)$  is the probability that a unemployed worker with residence in  $k$  at time  $t$  moves his residence to  $j$  at time  $(t+1)$ ;

$p_{kj}^i(t), i>0$  is the probability that a job in  $j$  occupied at time  $t$  by a worker resident in  $k$  is occupied at time  $(t+1)$  by a worker resident in  $j$ ;

$(1 - e^{-\phi_j})$  is the probability that a unemployed worker occupies a job in  $j$ .  $\phi_j$  is the potential demand of a job in  $j$  in a unit of time.

The main idea is to consider a job in a given zone (zone  $i$ ) and compute the two kinds of mobility: job and residential changes of employees and unemployees.

In accordance with what was previously said on the dependence of the transition probabilities on exogenous and endogenous factors:

$q_{kj}^i$  and  $p_{kj}^i$  can be written as follows:

$$q_{kj}^i = \frac{\lambda W_j(t) A_j Y_j(t) f_{ij}}{\tau_{ik}(t)}, \quad i > 0 \quad (8)$$

$$q_{kj}^o = \frac{\lambda W_j(t) A_j Y_j(t)}{\tau_{ok}(t)}$$

$$p_{kj}^i = \frac{\mu P_{oj}(t) A_j Y_j(t) Z_j(t) f_{ij}}{\tau_{ik}(t)}, \quad i > 0 \quad (9)$$

where:

$\lambda > 0$  parameter of speed in residential moving (to be calibrated);

$\mu > 0$  parameter of speed in worker substitution by a job (to be calibrated);

$f_{ij}$  deterrence function in moving from  $i$  to  $j$ ;

$W_j(t)$  stock of vacant dwellings in  $j$  at time  $t$ ;

$A_j$  accessibility to services (kept constant over time for the moment);

$$Y_j(t) = e^{-\beta r_j(t)} \quad (10)$$

where  $r_j(t)$  average price of a house in  $j$  at time  $t$  and  $\beta$  suitable parameter (to be calibrated);

$$Z_j(t) = e^{-\beta y_j(t)} \quad (11)$$

where  $y_j(t)$  average disposable income in  $j$  at time  $t$ ;

$$\tau_{ik}(t) = \sum_j A_j Y_j(t) f_{ij} [\lambda W_j(t)] + \mu P_{oj}(t) Z_j(t) + A_k Y_k(t) f_{ik} [1 + Z_k(t)], \quad i > 0; \quad (12)$$

$$\tau_{ok}(t) = \lambda \sum_j W_j(t) A_j Y_j(t) + A_k Y_k(t). \quad (13)$$



## 5.2. Stock dynamics

The problem of modelling the dynamics of physical stock arises in two subsystems: the housing market and the service sector.

Here we shall leave aside the question of residential stock, mainly because it involves a relatively slow process of change, but also because the model proposed is a simple accounting scheme describing house building activity i.e. new construction and demolitions. It can be considered a very simple case of the master equations approach already described (birth and death processes).

Of greater interest is the question of the service stocks, firstly because this is a phenomenon which causes relatively fast physical change in an urban area and secondly because the service sector is more sensitive than others to changes in demand.

To model the service stock we use the equations proposed by Harris and Wilson (1978):

$$W_j = \lambda \{D_j(W) - C_j(W_j)\} f(W_j) \quad (14)$$

where:

$W_j$  is the service stock (e.g. shopping centres) in location  $j$ ;

$C_j(W_j)$  is the cost associated with the supply of a stock of services  $W_j$  in  $j$ ;

$D_j(W) = \sum_i T_{ij}(W)$  is the total demand attracted in  $j$ , expressed in terms of total expenditure (i.e.  $D_j$  is the total revenue from services in  $j$  expressed in monetary terms);

$$T_{ij}(W) = Q_i \frac{W_j^\alpha e^{-\beta c_{ij}}}{\sum_j W_j^\alpha e^{-\beta c_{ij}}} \quad \text{is the total expenditure of residents in } i \text{ for services in } j; \quad (15)$$

$Q_i$  is the total expenditure of residents in  $i$ ;

$c_{ij}$  is the cost of transport between  $i$  and  $j$ ;

$\lambda, \alpha, \beta$  are non-negative parameters.

The function  $f(W_j)$ , assumed to be non negative and increasing, is a pick up factor which expresses the fact that the speed of the process described in (14) increases with the growth of the dimension of existing services in  $j$ .

In many applications, the following simplifying hypothesis are introduced:

$$f(W_j) = W_j$$

$$C_j(W_j) = k_j W_j, \quad k_j > 0 \text{ constant.}$$

Apart from the pick up factor  $f(W_j)$ , equation (14) basically postulates a growth rate of the dimension of services in each zone proportional to net profit i.e. the difference between revenue and cost. When this is positive there is an increase, when it is negative a decrease.

It is important to note that certain parameters of this model, in particular the exponent  $\alpha$  which appears in (15) have critical values which can result in bifurcations and structural changes in the solution.

It should also be pointed out that (15) defines a Logit-type model ensuring therefore coherence between the assumptions made about the behaviour of demand in mobility processes [cfr.: eq. (5)] and the service demand model.

The model of the service stock dynamics however cannot be used as it stands in (15) in conjunction with residential mobility as the latter is expressed in discrete time. Therefore it is necessary to express the stock model in discrete time as well. The proposed model modifies the continuous time version in such a way as to make the

treatment of prices and costs comparable with their treatment in the other markets (jobs and residential) considered in the general model:

$$W_j(t+1) = W_j(t) \exp \{ \epsilon [F_j(t) - k_j(t)] \} \quad (16)$$

where:

$F_j(t)$  is the revenue at time  $t$  per unit of service supply;

$k_j(t)$  is the cost at time  $t$  per unit of service supply;

with the definitions:

$$F_j(t) = D_j(t) / W_j(t) \quad (17)$$

$$D_j(t) = \sum_i G_i(t) \frac{[W_j(t)]^\alpha e^{-\beta(c_{ij} + x_j)}}{\sum_j [W_j(t)]^\alpha e^{-\beta(c_{ij} + x_j)}} X_j \quad \text{total revenue in } j \quad (18)$$

$$G_i(t) = \varphi_1 \sum_k P_{ki}(t) + \varphi_2 \sum_k P_{ik}(t) \quad \text{total demand from } i \quad (19)$$

$\varphi_1$  and  $\varphi_2$  being the frequency of service trips from places of residence and work, respectively, and  $x_j$  being the prices of services in  $j$ .

### 5.3. Price formation mechanisms

The price mechanisms (at the present state of development of the model) appear in the housing market and job market subsystems.

In relation to prices too it is possible to identify a close analo-



gy between the housing and the job markets. We illustrate here the case of the housing market.

There are two alternative ways of operating the model. The first corresponds to the hypothesis that the price adjustment mechanism is extremely fast, so fast in fact that it is possible to ignore the transitory fluctuations and assume that there is a state of equilibrium at all times. The second corresponds to a process of slow adjustment.

In the first case prices are determined as the solution of a system of market clearing equations, maintaining however the possibility that demand and supply are not perfectly balanced i.e. that a certain number of houses remain vacant (Anas, 1982; Leonardi, 1984):

$$D_j(r_1, \dots, r_n) = Q_j H_j(r_1, \dots, r_n) \quad (20)$$

where:

$r_j$  is the average price of a house in  $j$ ;

$D_j(r_1, \dots, r_n)$  is the demand for a house in  $j$  at a given time, expressed in function of the price;

$Q_j$  is the stock of houses available in  $j$  at a given time;

$H_j(r_1, \dots, r_n)$  is the probability that a house in  $j$  is requested by at least one unit of demand.

The demand function is given by a Logit type model:

$$D_j = \sum_i P_i p_{ij} = \sum_{i=1}^m P_i \frac{Q_j e^{\beta(c_{ij} + r_j)}}{\sum_{k=1}^n Q_k e^{\beta(c_{ik} + r_k)}} \quad (21)$$

with  $P_i$  being the resident population  $i$  and  $c_{ij}$  the cost of transport between  $i$  and  $j$ .

The supply function derived by probabilistic considerations at the micro level (Bertuglia et al., 1985) is:

$$H_j = (1 - e^{-\psi_j}) \quad (22)$$

with

$$\psi_j = \sum_{i=1}^m P_i \frac{e^{-\beta c_{ij}}}{\sum_{k=1}^n Q_k e^{-\beta(c_{ik} - r_k)}} \quad (23)$$

From (22) it follows that:

$$e^{-\psi_j} = 1 - \frac{\sum_{i=1}^m P_i p_{ij}}{Q_j} \quad (24)$$

is the probability that a house in  $j$  will remain vacant.

In the second case it is assumed that the price adjustment mechanism is slow enough to require explicit dynamic treatment. Here, it has been shown (Bertuglia et al., 1985) that the analogous dynamic of equilibrium equations (20) is given by the differential equations:

$$(1 - t\beta r_j) D_j(r_1, \dots, r_n) = Q_j H_j(r_1, \dots, r_n). \quad (25)$$

## 6. CONCLUSIONS

The Lowry model (1964), in its simplicity, was able to capture the basic essence of urban structure from a static viewpoint. The validity of this model has been demonstrated by a large number of empirical tests and applications whenever planners have been interested in evaluating the final goal and not the trajectory the urban system follows to reach it.

It is the aim of this study to deal with some of the essential aspects of the medium term dynamics of an urban area, such as population mobility, structural changes in services and market interactions.

Future research will obviously be necessary in order to improve some shortcomings of the present model system. Certain exogenous assumptions for the variables should be made endogenous and certain simplifying assumptions regarding the behaviour of some of the actors in the urban system be relaxed.

These can certainly be achieved developing suitable techniques thus increasing the realism of the model without altering its structure.

There is one important point to emphasize however in relation to future research. If decision-makers wish to evaluate not only the final result but the evolution the system goes through in reaching its equilibrium state appropriate cost-benefit techniques for the evaluation of alternative trajectories must be developed.

The aim should be at the very least to construct a series of performance indicators evaluating the output of the model. More ambitiously the aim should be to construct optimal control models for



which the present model system would act as an input. It should be stressed however that future research needs to move towards the development of methods and techniques of evaluation and optimisation suitable for a dynamic urban system.

This in fact will be the objective of the third phase of the I.R.E.S. "Location-Transport Interrelations" project.

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